

4. Successions

1. $p_0(\varepsilon) = 1 + E\left(\frac{1}{\varepsilon} - 2\right).$

2. $p_0(k) = 1 + E(k + \sqrt{k^2 - k}).$

3. $\lim \frac{n^2 + 4}{3n^2 - n} = \frac{1}{3}; \quad \lim \frac{n + 3}{n^3 - 6} = 0; \quad \lim \frac{n^4 + n^2}{n^3 - n + 27} = +\infty.$

4. $\lim \sqrt{(n-1)(n+7)} - n = 3; \quad \lim \frac{\sqrt{n^2 + 1} + \sqrt{n}}{\sqrt[4]{n^3 + n} - \sqrt{n}} = +\infty;$
 $\lim(\sqrt{n^2 + \sqrt{n}} - n)(\sqrt{n+1} + \sqrt{2n}) = \frac{1 + \sqrt{2}}{2}.$

5. $\lim \left(\frac{n+2}{n+1}\right)^{n+3} = e; \quad \lim \left(1 + \frac{2}{n}\right)^{7n} = e^{14}; \quad \lim \left(\frac{5n+2}{15n-4}\right)^{\frac{3n+4}{9n-6}} = \left(\frac{1}{3}\right)^{1/3}.$

6. $\lim \sqrt[n]{2} = 1; \quad \lim \sqrt[n]{n^2 - 7} = 1; \quad \lim \sqrt[n]{\ln n} = 1; \quad \lim \sqrt[n]{\frac{n^2 + 1}{n^7 - 2}} = 1.$

7. $\lim \frac{1^2 + 2^2 + \dots + n^2}{n^3} = \lim \frac{n^2}{n^3 - (n-1)^3} = 1/3; \quad \lim \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n} =$
 $\lim \frac{\frac{1}{n}}{n - (n-1)} = 0; \quad \lim \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{(n+1)\sqrt{n}} = \lim \frac{\sqrt{n}}{(n+1)\sqrt{n} - n\sqrt{n-1}} = 2/3.$

8. $\lim \frac{1! + 2! + \dots + n!}{(n+2)!} = \lim \frac{n!}{(n+2)! - (n+1)!} = \lim \frac{1}{(n+1)(n+1)} = 0;$
 $\lim \frac{\sin \alpha + 2^2 \sin \frac{\alpha}{2} + \dots + n^2 \sin \frac{\alpha}{n}}{n^2} = \lim \frac{n^2 \sin \frac{\alpha}{n}}{n^2 - (n-1)^2} = \frac{\alpha}{2}.$

9. $\lim \left(\frac{n+a}{n+2}\right)^{an+b} = e^{a(a-2)}; \quad \lim \left(\frac{n+b}{n+1}\right)^{2n+a} = e^{2(b-1)}; \quad \text{alshores}$
 $e^{a(a-2)} = e^{2(b-1)} \Rightarrow a(a-2) = 2(b-1).$

10. $a > b, \quad \lim \frac{a^n - b^n}{a^n + b^n} = 1; \quad a < b, \quad \lim \frac{a^n - b^n}{a^n + b^n} = -1.$

11. -

12. -

13. $\lim a_n = (2, 1); \quad \lim b_n = (0, 0); \quad \lim c_n = (0, 2); \quad \lim d_n = (1, 0, 2); \quad \lim e_n \text{ no existeix}$
 doncs $\frac{n^2 + 3}{n} \rightarrow +\infty; \quad \lim f_n = (1/4, 1, 0).$

14. -

15. -

16. -