# On the Goldberg Conjecture

Kouei Sekigawa

*Niigata University*

Concerning the integrability of almost Kaehler manifolds, there is a long-standing conjecture by S.I.Goldberg (1969) saying that a compact alomst Kaehler Einstein manifold is integrable (and hence, Kaehler Einstein). The conjecture is true in the case where the scalar curvature is non-negative (1987). However, the conjecture is still open in the case where the scalar curvature is negative. It is notable that non-compact complete counter examples to the conjecture are given by Apostolov, Draghici and Moroianu and also that an indefinite counter example is given by Matsushita. In the present talk, we introduce several partial affirmative answers to the conjecture and some related topics related to the conjecture.

References

[1] Apostolov,V.,Draghici,T.,Moroianu,A.: A splitting theorem for Kaehler manifolds whose Ricci tensors have constant values, Intern.J.Math.,12,769-789(2001).

[2] Apostolov,V.,Gauduchon,P.,Calderbank,D.M.J.: The geometry of weakly selfdual Kaehler surfaces. Compositio Math.,135,279-322(2003).

[3] Goldberg,S.I.: Integrability of almost Kaehler manifolds. Proc.Amer.Math.Soc.,21,96-100(1969).

[4] Nurowski,P.,Przanowski,M.: A four-dimensional example of Ricci flat metric admitting almost Kaehler non-Kaehler structure. Classical Quantam Gravity,16, L9-L13(1999).

[5] Oguro,T.,Sekigawa,K.: Notes on the Goldberg Conjecture in Dimension Four, Complex,Contact and Symmetric Manifolds, In Honor of L.Vanhecke, Birkhauser,

[6] Sekigawa, K.: On some compact Einstein almost Kaehler manifolds. J.Math.Soc.Japan.,36,677-684(1987).