Breakdown of heteroclinic orbits for some analytic unfoldings of the Hopf-zero singularity

I. Baldomá[1] * T.M. Seara[2] †

12th September 2005

[1] Departament de Matemàtica Aplicada i Anàlisi
Universitat de Barcelona
Gran Via 587, 08007 Barcelona, Spain

[2] Departament de Matemàtica Aplicada I
Universitat Politècnica de Catalunya
Diagonal 647, 08028 Barcelona, Spain

Summary

In this paper we study the exponentially small splitting of a heteroclinic orbit in some unfoldings of the central singularity also called Hopf-zero singularity.

The fields under consideration are of the form:

$$\begin{aligned}
\frac{dx}{d\tau} &= -\delta xz - y\,(\alpha + c\delta z) + \delta^{p+1} f(\delta x, \delta y, \delta z, \delta) \\
\frac{dy}{d\tau} &= -\delta yz + x\,(\alpha + c\delta z) + \delta^{p+1} g(\delta x, \delta y, \delta z, \delta) \\
\frac{dz}{d\tau} &= \delta\left(-1 + b(x^2+y^2) + z^2\right) + \delta^{p+1} h(\delta x, \delta y, \delta z, \delta),
\end{aligned}$$

*barraca@mat.ub.es
†tere.m-seara@upc.edu

where f, g and h are real analytic functions, α, b and c are constants and δ is a small parameter.

When $f = g = h = 0$ the system has a heteroclinic orbit between the critical points $(0, 0, \pm 1)$ given by: $\{(x, y) = (0, 0)\ ; -1 < z < 1\}$.

Let $d^{\mathrm{s,u}}$ be the distance between the one dimensional stable and unstable manifold of the perturbed system measured at the plane $z = 0$. We prove that for any f, g such that $\hat{m}(\mathrm{i}\,\alpha) \neq 0$, where $\hat{m}$ is the Borel transform of the function $m(u) = u^{1+\mathrm{i}\,c}(f + \mathrm{i}\,g)(0, 0, u, 0)$

$$|d^{\mathrm{s,u}}| = 2\pi\,\mathrm{e}^{c\pi/2}\,|\hat{m}(\mathrm{i}\,\alpha)|\delta^{p}\,\mathrm{e}^{-\pi|\alpha|/(2\delta)}(1 + O(\delta^{p+2}|\log\delta|)), \quad p > -2.$$

Keywords: Exponentially small splitting, Hopf-zero bifurcation, Melnikov function, Borel transform.